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Photon as the zero-mass limit of DKP field

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Abstract. It is shown that Maxwell's equations can be obtained as the zero-mass limit of the Duffin–Kemmer–Petiau equation for the massive spin-one particle with appropriate identification of the components of the DKP spinor with electromagnetic field strengths and potentials. The electromagnetic Lagrangian also follows from the DKP Lagrangian in the same limit.

Passage from the classical wave theory of light to quantum mechanics is usually done (Good 1957) by writing Maxwell's equations in Schrödinger form and then replacing the operator ∇ by $(i/\hbar)\boldsymbol{p}$ so as to extract therefrom the particle aspects of the electromagnetic field. If complex combinations of the electric and magnetic fields are taken as the elements of a three-component spinor, Maxwell's curl equations can be synthesised into a form similar to that of the Weyl equation for the neutrino. In this three-component formulation, the divergence equations are imposed as constraint equations. Further, these are valid only in free space in the absence of any source. These two deficiencies are rectified in the work of Moses (1958, 1959), who found a four-component spinor formulation to cast the Maxwell equations in the form of a massless Dirac equation. He incorporated the source in the form of a four-component spinor and synthesised the four Maxwell equations in the presence of a source. In neither of these two approaches do the equations seem to follow from an Euler–Lagrange variational principle.

Much earlier than the above attempts, Duffin (1938), Kemmer (1939) and Petiau (1938) had formulated the wave equation for massive spin-one particles. Since the photon is a spin-one particle, one would expect that the Maxwell equations can be obtained from the DKP equation in the limit of zero mass. Attempts in the past to realise this seem not to have succeeded; an equation somewhat different from the massless DKP equation has been used (Corson 1953, Bludman 1957) to describe the photon. It is the purpose of this paper to show that, with suitable identification of the components of the DKP wavefunction with electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ and the potentials $\boldsymbol{A}$ and ϕ, the DKP equation goes over to Maxwell's equations in the limit of zero mass.

The DKP equation for a particle of mass m and spin one is

$$(\beta_\mu \partial_\mu + m)\psi = 0 \tag{1}$$

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where the β's are 10×10 matrices. We choose the ten-component DKP spinor as

$$\psi=\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{B} \\ -m\boldsymbol{A} \\ -m\phi \end{pmatrix} \tag{2}$$

where the three vectors $\boldsymbol{E}$, $\boldsymbol{B}$ and $-m\boldsymbol{A}$ stand for the three-component columns and $-m\phi$ is the tenth component. With this choice, the explicit form of equation (1) becomes

$$\left\{\left[\begin{array}{ccc|ccc|ccc|c}
0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{i}\partial_4 & 0 & 0 & -\partial_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{i}\partial_4 & 0 & -\partial_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{i}\partial_4 & -\partial_3 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\partial_3 & \partial_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \partial_3 & 0 & -\partial_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\partial_2 & \partial_1 & 0 & 0 \\
\hline
\mathrm{i}\partial_4 & 0 & 0 & 0 & \partial_3 & -\partial_2 & 0 & 0 & 0 & 0 \\
0 & \mathrm{i}\partial_4 & 0 & -\partial_3 & 0 & \partial_1 & 0 & 0 & 0 & 0 \\
0 & 0 & \mathrm{i}\partial_4 & \partial_2 & -\partial_1 & 0 & 0 & 0 & 0 & 0 \\
\hline
-\partial_1 & -\partial_2 & -\partial_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] + mI\right\}
\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ B_1 \\ B_2 \\ B_3 \\ -mA_1 \\ -mA_2 \\ -mA_3 \\ -m\phi \end{pmatrix} = 0. \tag{3a}$$

Here I stands for the 10×10 unit matrix. The equation (3a) can be written in a compact notation as

$$\begin{pmatrix} m & 0 & -\partial/\partial t & -\boldsymbol{\nabla}\,. \\ 0 & m & \boldsymbol{\nabla}\times & 0 \\ \partial/\partial t & -\boldsymbol{\nabla}\times & m & 0 \\ -\boldsymbol{\nabla}\,. & 0 & 0 & m \end{pmatrix}\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{B} \\ -m\boldsymbol{A} \\ -m\phi \end{pmatrix}=0 \tag{3b}$$

which when written componentwise reads

$$\boldsymbol{E}=-\boldsymbol{\nabla}\phi-\partial\boldsymbol{A}/\partial t \tag{4a}$$

$$\boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A} \tag{4b}$$

$$\boldsymbol{\nabla}\times\boldsymbol{B}-\partial\boldsymbol{E}/\partial t+m^2\boldsymbol{A}=0 \tag{4c}$$

$$\boldsymbol{\nabla}\,.\,\boldsymbol{E}+m^2\phi=0. \tag{4d}$$

Equations (4a) and (4b) give the definitions of $\boldsymbol{E}$ and $\boldsymbol{B}$ in terms of the potentials $\boldsymbol{A}$ and ϕ. The two homogeneous Maxwell equations

$$\boldsymbol{\nabla}\,.\,\boldsymbol{B}=0 \qquad \text{and} \qquad \boldsymbol{\nabla}\times\boldsymbol{E}+\partial\boldsymbol{B}/\partial t=0$$

follow directly from them. Equations (4c) and (4d), in the limit $m\to 0$, give the two inhomogeneous Maxwell equations in free space.

In order to describe the situation in a material medium and in the presence of a source, a source spinor J is introduced and the DKP equation reads

$$(\beta_\mu\,\partial_\mu+m)\psi+4\pi J=0 \tag{5}$$

where

$$\psi = \begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{H} \\ -m\boldsymbol{A} \\ -m\phi \end{pmatrix} \tag{6}$$

with

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi \boldsymbol{P} \qquad \text{and} \qquad \boldsymbol{H} = \boldsymbol{B} - 4\pi \boldsymbol{M}, \tag{7}$$

$\boldsymbol{P}$ and $\boldsymbol{M}$ being the electric polarisation and magnetisation of the medium respectively. The source spinor J is

$$J = \begin{pmatrix} -m\boldsymbol{P} \\ m\boldsymbol{M} \\ \boldsymbol{j} \\ \rho \end{pmatrix} \tag{8}$$

with $\boldsymbol{j}$ and ρ representing current and charge densities respectively.

The Lagrangian in the presence of sources and inside a medium can be written as

$$\mathscr{L} = -(1/4m)(\bar{\psi}\beta_\mu\, \partial_\mu \psi - \partial_\mu \bar{\psi}\beta_\mu \psi) - (2\pi/m)(\bar{\psi}J + \bar{J}\psi) - \tfrac{1}{2}\bar{\psi}\psi. \tag{9}$$

It is easily verified that equation (5) and its adjoint equation follow from a variational principle using the above Lagrangian.

The equation (5), written componentwise, gives

$$\boldsymbol{D} - 4\pi \boldsymbol{P} \equiv \boldsymbol{E} = -\boldsymbol{\nabla}\phi - \partial \boldsymbol{A}/\partial t \tag{10a}$$

$$\boldsymbol{H} + 4\pi \boldsymbol{M} \equiv \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{10b}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} - \partial \boldsymbol{D}/\partial t + m^2 \boldsymbol{A} = 4\pi \boldsymbol{j} \tag{10c}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} + m^2 \phi = 4\pi\rho \tag{10d}$$

of which (10a) and (10b) relate the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ to the potentials as in the source- and medium-free case. The other two equations in the limit $m \to 0$ are seen to be the inhomogeneous Maxwell equations in a material medium in the presence of sources.

The Lagrangian (9), in terms of field strengths and potentials, reads

$$\begin{aligned} \mathscr{L} = {} & -\tfrac{1}{2}\boldsymbol{D} \cdot (\dot{\boldsymbol{A}} + \boldsymbol{\nabla}\phi) - \tfrac{1}{2}\boldsymbol{H} \cdot (\boldsymbol{\nabla} \times \boldsymbol{A}) + \tfrac{1}{2}\boldsymbol{A} \cdot (\dot{\boldsymbol{D}} - \boldsymbol{\nabla} \times \boldsymbol{H}) + \tfrac{1}{2}\phi \boldsymbol{\nabla} \cdot \boldsymbol{D} \\ & + 4\pi(\boldsymbol{D} \cdot \boldsymbol{P} + \boldsymbol{H} \cdot \boldsymbol{M} + \boldsymbol{j} \cdot \boldsymbol{A} - \rho\phi) - \tfrac{1}{2}(\boldsymbol{D}^2 - \boldsymbol{H}^2) - \tfrac{1}{2}m^2(\boldsymbol{A}^2 - \phi^2) \end{aligned} \tag{11}$$

which in free space and in the massless limit reduces to

$$\mathscr{L} = \tfrac{1}{2}\boldsymbol{A} \cdot (\dot{\boldsymbol{E}} - \boldsymbol{\nabla} \times \boldsymbol{B}) + \tfrac{1}{2}\phi \boldsymbol{\nabla} \cdot \boldsymbol{E}. \tag{12}$$

By using the Maxwell field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for $\boldsymbol{E}$ and $\boldsymbol{B}$, the Lagrangian (12) becomes

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu F^{\mu\nu})A_\nu \tag{13}$$

and, on throwing away a four-divergence term, it reduces to the familiar form of the electromagnetic Lagrangian

$$\mathscr{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{14}$$

A remark on the Lagrangian in equation (9) is in order. On the face of it, the first two terms of the Lagrangian appear to be singular in the zero-mass limit. This situation does not arise in the standard DKP Lagrangian. But here, from physical considerations, we have chosen ψ in the form given by equation (6), which when used in the standard DKP Lagrangian gives an extra mass dimension to the Lagrangian density. This necessitates the division by mass. However, this mass factor exactly cancels out in the explicit expression of the Lagrangian as given by equation (11), and hence no problem arises in going to the zero-mass limit.

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